

Parametric investigation of vibration of stiffened structural steel plates using finite element analysis and grey relational analysis

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ABSTRACT

Thin plates with arbitrary shapes and stiffeners find wide usage in construction, aerospace, marine, etc. industries. In the present work a parametric investigation of thin structural steel plates has been carried out. The design parameters considered were the subtended angle and aspect ratio of plates to account for the different shapes of plates used for various applications. Another design parameter that was considered is the stiffener and its different orientation. The impact of varying parameters on the first five modal frequencies and in turn the stiffness was considered as the response. Finite element method (FEM) coupled with Taguchi's L_{16} orthogonal array and grey relational analysis was used to maximize the first five natural frequencies simultaneously. The thin plate was subjected to modal analysis when all its sides are in simply supported boundary condition. The optimum combination of design variables predicted by grey relational analysis is a thin plate with 80° subtended angle, 1.75:1 aspect ratio and crossed stiffener orientation for simultaneous maximization of the frequencies. Analysis of variance revealed highest contribution of stiffener type which affects the first 5 natural frequencies simultaneously.

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1. Introduction

Metal plates are used in many fields. Some applicable sectors are namely aerospace, structural constructions, electrical and electronics, marine, military & weapons and automobile industries. Bare plates by themselves have relatively low stiffness. By adding another element to it we can increase its stiffness. Such an element is known as a 'stiffener'. Several works have previously been done in the field of free vibration of plates using finite element analysis (FEA) (Isanaka et al., 2020; Tarn et al., 1988; Alinia, 2005; Tayşi, 2010; Cunha et al., 2018). Isanaka *et al.* (2020) used FEA model for free vibrational investigation, in which multiple regular shaped plates were analysed. Tarn *et al.* (1988) examined the stability of stiffened plates by utilizing the 'finite strip method'. The hardened plate was demonstrated as an arrangement of stiffener-plate components and bar components, including twist just as bending effects. Alinia (2005) showed that by adding stiffener to

a bare rectangular plate the shear stress required to deform the plate increases. The increment can be altered by altering aspect ratio, number and design of stiffeners. Tayşi *et al.* (2010) proposed that supported-constraint boundary condition gives maximum buckling loads, which meant critical buckling load increases as the support on the given plate increases. Through the application of computational modelling along with constructural design it is possible to make recommendations about the suitable geometry to be used for plates (2018).

Different techniques have been employed to optimise the plate stiffener assembly (Cunha *et al.*, 2019; Kalassy & Marcelin, 1997; Putra *et al.*, 2019; Mizusawa *et al.*, 1979; Li *et al.*, 2013). Cunha *et al.* (2019) performed an investigation of a Genetic Algorithm (GA) based mathematical optimization of stiffener-plates assembly exposed to transverse loads. The constructural design technique was utilized to characterize the search space, pointing to limit the deflection of the assembly, while keeping the aggregate material volume steady. A similar study on stiffener optimisation using GA was done by Kallassy *et al.* (1997). Putra *et al.* (2019) optimized stiffened plates used in ships using hybrid GA technique, while Mizusawa *et al.* (1979) performed the free vibration of thin plates by the Rayleigh-Ritz technique. Li *et al.* (2013) optimized the topology to create the stiffener layout for stiffener-plate assembly.

From the above discussion, it may be observed that a significant attention has been given to optimization of geometry and stiffness of plates. However, research works that would deal with varying subtended angle, stiffener type and plate’s aspect ratio simultaneously and then optimising to get the best combination are still missing. The major objective of this paper is to find how the parameters affect the overall stiffness of the assembly, and ultimately to find out the best possible combination of the three factors using grey relational analysis (GRA). GRA method has been used in optimization of numerous industrial processes (Ho *et al.*, 2003). The unique feature of GRA is that the optimization may be carried out using the results at discrete points and are therefore especially beneficial in the present work.

2. Methodology

2.1. Modelling approach

A total of three design variables were varied at four levels given in Table 1. The stiffener arrangements are illustrated in Figure 1. Initially the assembly was modelled with stiffener as an integrated part of the plate and then later they were modelled separately and welded with the help of Solidworks. In this study, the first five frequencies of all the stiffener assemblies were obtained using ANSYS for all the modes. The goal is to obtain the best design to simultaneously maximize the first five natural frequencies.

2.2. Dimensions

Equal area is maintained for all the plates. The various factors that were varied are subtended angle, aspect ratio and stiffener type. The material selected for both plate and stiffener are structural steel, as it has many industrial applications. The properties of plate and stiffener and dimensions of the plate are as shown in Table 2 and Table 3 respectively and are based on practicality and other research work (Isanaka *et al.*, 2020).

Table 1. Important parameters for stiffener-plate assembly

Variables	Units	Symbols	Levels			
			1	2	3	4
Subtended Angle	°	A	70*	75	80	85
Aspect Ratio	-	B	1*	1.25	1.5	1.75
Stiffener Type	-	C	Bare*	Single Stiffener	Crossed	Diagonally crossed

*Initial parametric combination

3. Results and Discussion

3.1. Analysis of simulation results

The simulation was carried out initially by considering the stiffener as an integral part of the plate i.e., the stiffener wasn’t bonded. The combination of parameters given in Table 1 considered for simulations was as per Taguchi’s L₁₆ array. Thereafter, the analysis on stiffener as a welded part (without mesh refinement) was

done. There was a difference of ~5% in the difference of absolute values of all the frequencies. Furthermore, mesh refining was carried out for the welded part and the observations are recorded in Table 4. After the model is modified to mimic real life scenario (bonded analysis), the element size of mesh is further reduced to 2.0×10^{-4} m and finally quadrilateral elements are used. Further reduction in element size didn't yield significant change in the results but costs computational time. The average percentage change in results of non-bonded analysis and bonded analysis (after mesh refinement) was seen to be ~7.23%. Figures 2-4 shows the various modes obtained with reference to Table 4 for the stiffened plates (excluding bare plates).

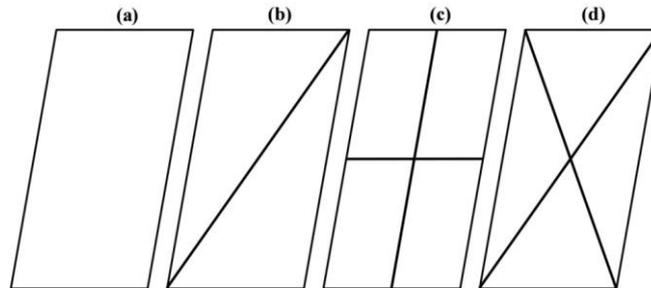


Figure 1. Various stiffener arrangements used (a) bare (b) single stiffened (c) cross stiffened and (d) diagonally crossed stiffened

Table 2. Plate and stiffener properties considered

Plate				
Support condition	Plate area	Plate thickness	Modulus of Elasticity	Poisson's Ratio
Simply Supported	197.12 mm ²	0.254 mm	1.1721×10^5 N/mm ²	0.3
Stiffener				
Dimensions		Modulus of Elasticity		Poisson's Ratio
2.54 mm × 0.254 mm		1.1721×10^5 N/mm ²		0.3

Table 3. Plate dimensions for various cases

Aspect ratio	1	1.25	1.5	1.75
Length (mm)	14.48	16.19	17.75	18.84
Breadth (mm)	14.48	12.95	11.65	10.97

Table 4. Frequencies for first five modes considering bonded stiffener plate assembly after mesh refinement and grey relational grade

Sl. No.	Frequency 1 (Hz)	Frequency 2 (Hz)	Frequency 3 (Hz)	Frequency 4 (Hz)	Frequency 5 (Hz)	Grade
1	4954.8	11029	13433	17947	24338	0.3788
2	10118	12449	18771	20237	22094	0.4334
3	18185	18555	20057	22876	23147	0.6772
4	13898	19116	20159	21036	21084	0.5807
5	10489	11879	18842	19941	21670	0.4271
6	4987.4	10467	14210	18167	21436	0.3520
7	14173	19272	19786	20904	20996	0.5836
8	17953	18209	20731	23793	24268	0.6956
9	17978	19485	19636	20208	21127	0.6549
10	14490	19372	19591	20313	20409	0.5808
11	5177.1	9810.1	15916	17379	20760	0.3494
12	13412	16853	24553	26642	28027	0.8472

Sl. No.	Frequency 1 (Hz)	Frequency 2 (Hz)	Frequency 3 (Hz)	Frequency 4 (Hz)	Frequency 5 (Hz)	Grade
13	14814	19355	20155	20447	20577	0.5923
14	17752	18665	19921	21324	21568	0.6359
15	10727	12049	18692	19878	20944	0.4230
16	5484.2	9479.1	16154	17874	21834	0.3601

3.2. Optimization of simulation results using grey relational analysis

In this study, based on the number of factors and their levels, L₁₆ orthogonal array was used and the simulations were further processed for optimization using grey relational analysis. Initially pre-processing of data was carried out where the frequencies were normalized between 0 and 1. Since higher frequency is desirable (which would lead to higher stiffness), they were normalized following higher-the-better quality characteristics. After grey relational generation, the grey relation coefficient was computed. Finally, the grey relational grade which is a multi-performance indicator was calculated. The grey relational grades are also laid down in Table 4. The procedure of grey relational analysis and formula may be found in Ref. (Ho et al., 2003).

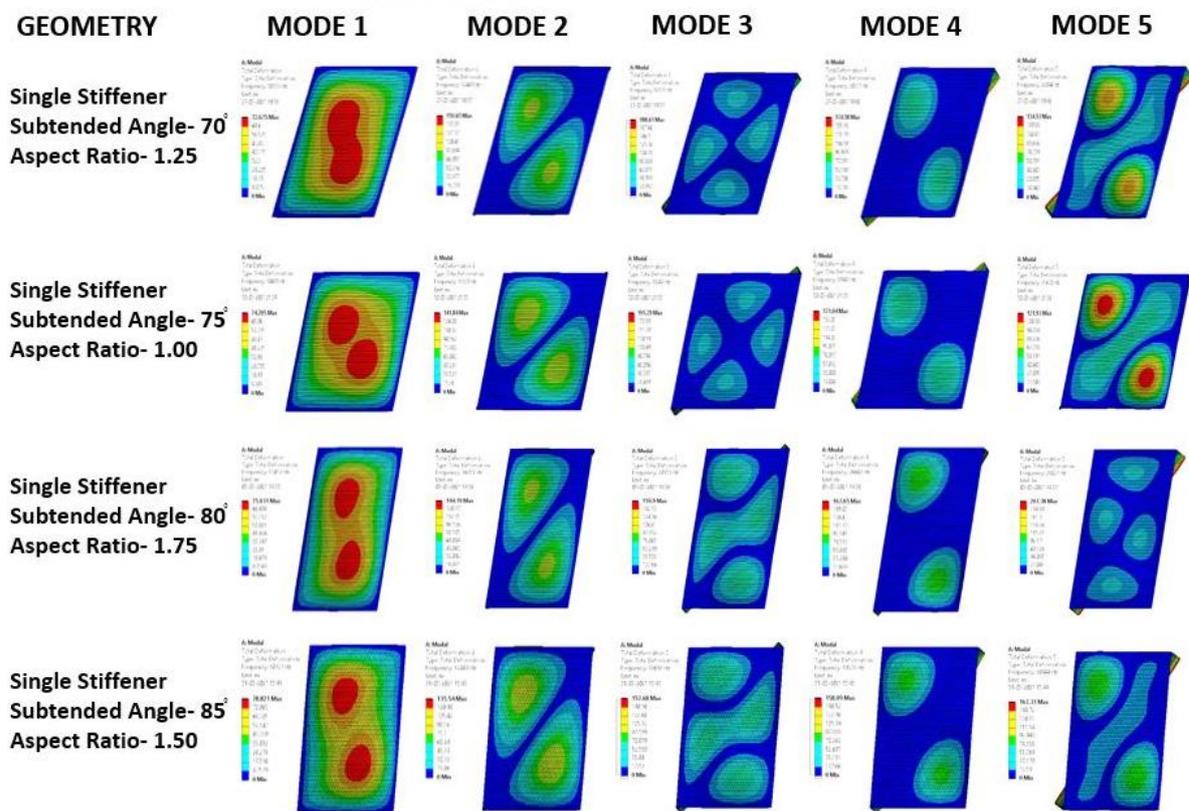


Figure 2. First five mode shapes of single stiffener plates

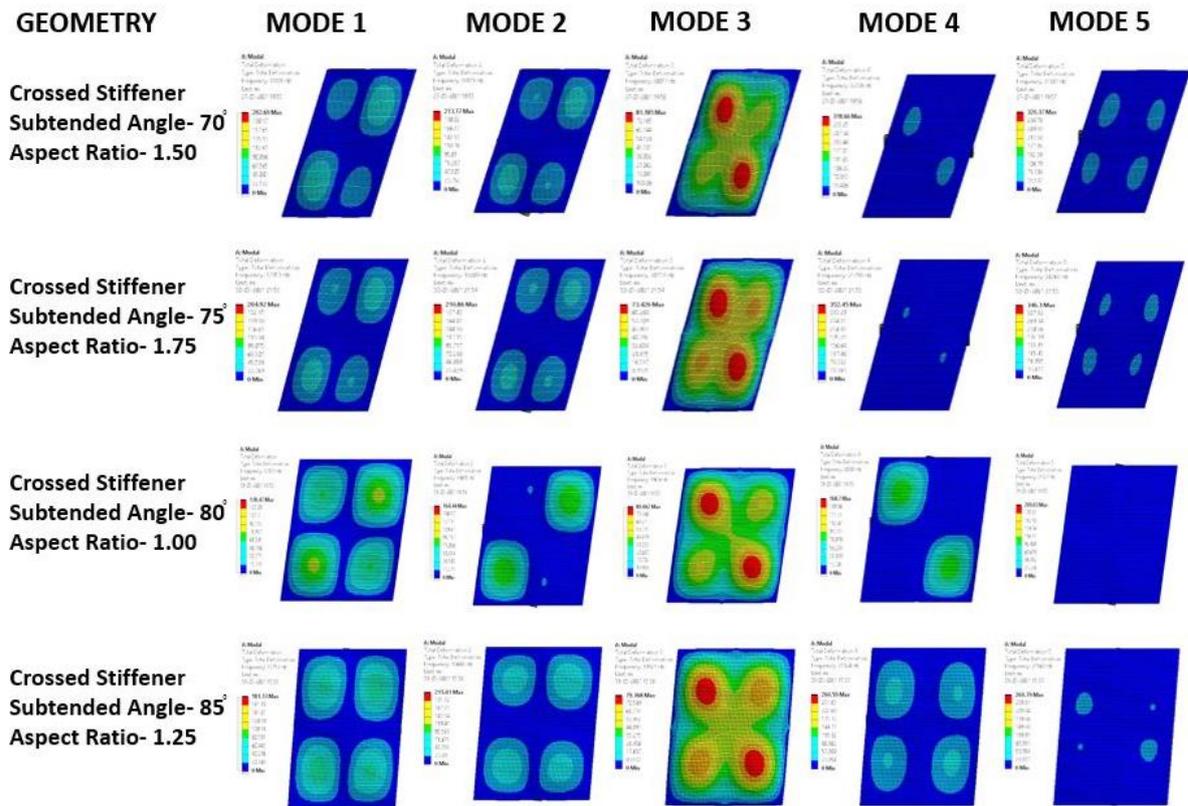


Figure 3. First five mode shapes of crossed stiffener plates

Finally, the optimal parametric combination was obtained from main effects plot given in Figure 5. The optimal combination predicted by grey relational analysis is A3B4C3 i.e., 80° subtended angle, 1.75 aspect ratio and cross-stiffened stiffener orientation. Next a confirmation test was carried out to compare the optimal grade with an initial test run. Here the initial test run considered is A1B1C1 i.e., bare plates with 70° subtended angle and aspect ratio 1. The results are shown for the validation run along with the simulated optimized results in Table 5. The corresponding mode shapes are shown in Figure 6. A noteworthy improvement in grade is achieved (84.74 %) at optimized subtended angle, aspect ratio and stiffener orientation. The grey relational grade at optimality condition can be also predicted from the formula given in Ref. (Ho et al., 2003). It can be seen from Table 5 that the grades of predicted and simulated result for optimal combination of parameters are in good accordance with each other. Analysis of variance (ANOVA) was also performed to reveal the significance of design parameters on the grade and the results are given in Table 6. The highest percent contribution is seen for the stiffener orientation followed by aspect ratio and subtended angle.

The mode shapes at optimized conditions in Figure 6 evidently shows that in every mode, the major deformation tends to take place in the stiffener rather than the plate itself, which is desired. It may be noted here that higher frequency is obtained at higher skewness and aspect ratio since the boundaries move more towards the centre of the plate (Isanaka et al., 2020). Though, the skewness and aspect ratio had lesser effect in increasing the overall stiffness of curvilinear stiffened plates (Isanaka et al., 2020). Topological optimization of stiffener arrangement based on nature inspired growth of leaf veins as well as shape optimization procedures lead to an increase in overall plate stiffness (Li et al., 2013). But the present work achieves higher frequency for first five modes using a more simplistic GRA.

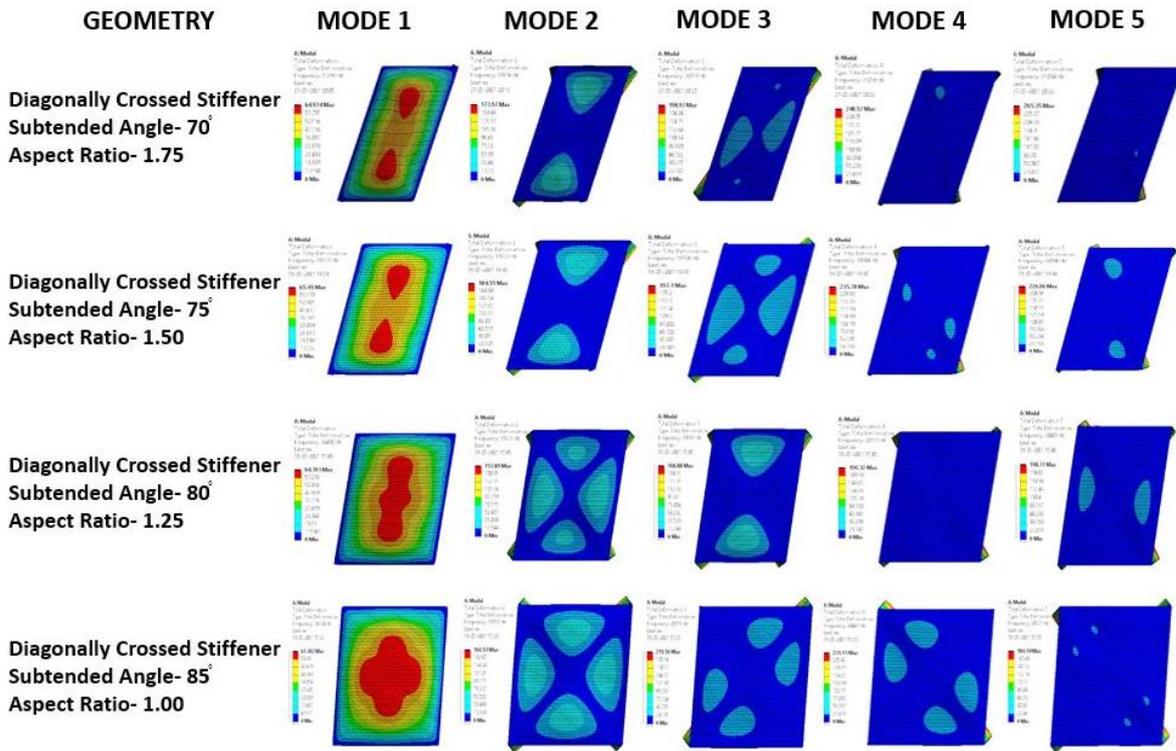


Figure 4. First five mode shapes of diagonally crossed stiffener plates

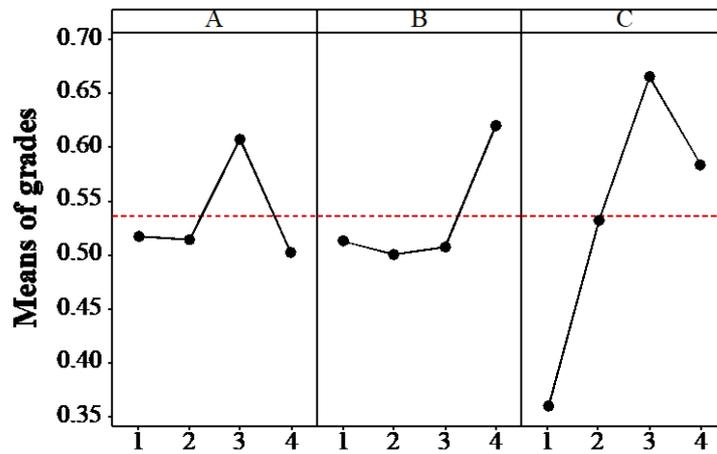


Figure 5. Main effects plot for grey relational grade

Table 5. Validation test results

	Initial	Optimal	
		Predicted	Simulated
Level	A ₁ B ₁ C ₁	A ₃ B ₄ C ₃	A ₃ B ₄ C ₃
Frequency 1 (Hz)	4954.8		17848
Frequency 2 (Hz)	11029		18139
Frequency 3 (Hz)	13433		20916
Frequency 4 (Hz)	17947		24009
Frequency 5 (Hz)	24338		24475
Grade	0.3788	0.8229	0.6998

Improvement in grade = 0.321 (84.74%)

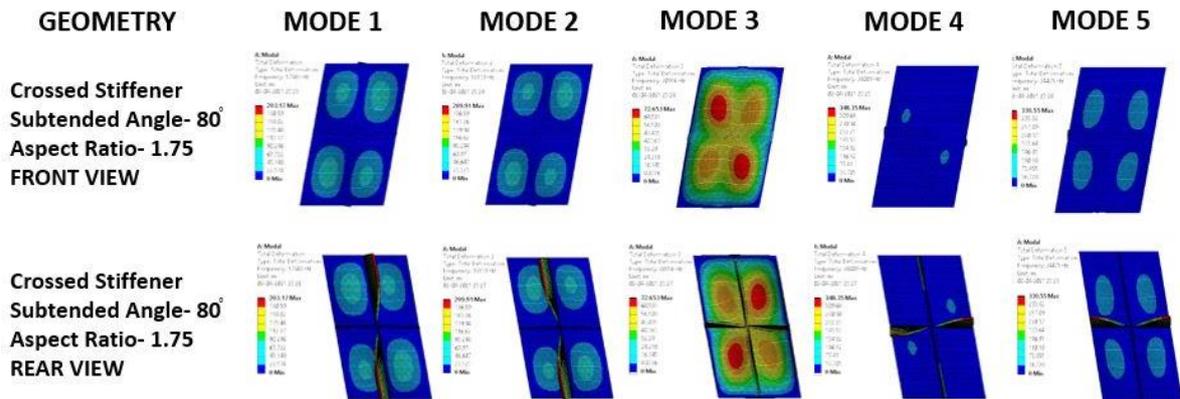


Figure 6. First five mode shapes of optimized stiffener plate assembly – front and rear view

Table 6. ANOVA results for grey relational analysis of stiffener-plate assembly

Source	DF	Adj. SS.	Adj. MS.	F-Value	% Contribution
Subtended Angle	3	0.02837	0.009457	0.84	8.46
Aspect Ratio	3	0.039	0.012999	1.16	11.63
Stiffener Type	3	0.20072	0.066905	5.97	59.86
Error	6	0.06724	0.011206		20.05
Total	15	0.33532			100

4. Conclusions

In this work, an ideal combination of plate's subtended angle, aspect ratio and stiffener's type were found for the stiffener-plate assembly by using the results of modal analysis in simply supported condition as a reference. The process was further optimised by the application of Taguchi's orthogonal array with GRA. The optimal combination comes out to be 80° subtended angle, 1.75 aspect ratio and crossed stiffener orientation (A3B4C3 combination). To confirm the correctness of the analysis, the best combination that resulted from the GRA was again subjected to modal analysis, which gave an improvement of ~85% in the overall system's grade with respect to the initial chosen condition. ANOVA results revealed that stiffener and its orientation have the highest influence in controlling the frequencies and in turn the stiffness of the plates. Further research work may be carried out for parametric investigations considering the number of stiffeners, other geometries of plates, various boundary conditions and optimize deflection of plates along with stiffness.

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