

## Comparison of robust optimal QFT controller with TFC and MFC controller in a multi-input multi-output system

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### ABSTRACT

This paper suggests a practical approach for the development of a stable robot controller using the Quantitative Feedback Principle (QFT). Robot manipulators have a multivariable nonlinear transfer function, the implementation of the QFT method includes, first the conversion of their nonlinear plant into a group of linear and uncertain plant set, and then an ideal robust controller for each set has been designed. To demonstrate the effectiveness of our algorithm, we show the implementation of the two degrees of freedom manipulator. In the approach provided, the controller has been designed directly by specifying and optimizing the transfer function coefficients using a genetic algorithm. The consistency and limitations of the method are considered to be the restrictions of the problem in the optimization process. System stability and tracking problem are perceived to be the limitations of the system in the optimization process. Non-linear simulations on the tracking problem are carried out and the results illustrate the performance of the controllers. Finally, the controller constructed based on the QFT approach is compared with the TFC and MFC (Fuzzy) controllers and it is shown that the QFT methodology indicates a controller that has increased control efficiency.

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## 1. Introduction

Most systems in the world, such as robots, have nonlinear dynamics that make it extremely difficult to control these systems. Robots are in the Lagrangian Dynamic System class, but some physical properties of them allow their straightforward control. A lot of focus has been paid in recent decades to control the operation of the robot, and a lot of work has been conducted to manipulate the robot. Various control mechanisms and devices have been introduced to robot links, each with its benefits and drawbacks (Luh, 1983), (Lewis, 1999 .) In this paper, to validate the proposed control method, we consider the combination of Robust control and GA on the two links of a robot with two nonlinear degrees of freedom, and as a basic control method, quantitative feedback theory (QFT) is used. Many real systems have a high degree of instability in the open-loop transfer functions, which makes it very difficult for the closed-loop system to have sufficient stability margins and good control efficiency. Therefore, in such systems, a single fixed controller is included in the "robust control" family. Quantitative Feedback Theory (QFT) is a robust feedback control system design. This methodology

developed by Horowitz (Horowitz, 1991), (Horowitz, 1992), enables the direct design of the closed-loop robust output and stability requirements. Simply QFT controller design approach can be described as follows: In parametric uncertain systems, we must first produce plant models before the QFT design (at a fixed frequency, the plant frequency response set is called a template). Provided plant models, QFT transforms the closed-loop magnitude specification to the nominal open-loop function (this is called the QFT bound). The nominal open-loop function is then configured to fulfill its constraints at the same time as ensuring nominal closed-loop stability. In the QFT system, the non-linear plant is transformed into a family of linear and unknown processes. For this aim, QFT literature provides a variety of methods (Horowitz, 1991), (Horowitz, 1992), Gharib et al. (2010), Gharib et al. (2011), (Gharib & Moavenian, 2012) including Linear Time-Invariant Equivalent (LTIE) of non-linear plants and Non-Linear Equivalent Disturbance Attenuation (NLEDA) techniques.

Also, in recent years, fuzzy logic controllers (FLC) have been widely used for industrial processes such as robots along with other control techniques such as Adaptive control or optimal control (Lian & Fu Lin 2005), Sliding-mode control (Utkin VI. (1977)), (Ching Chiou, K. (2005)), owing to their heuristic nature associated with simplicity and effectiveness for both linear and nonlinear systems. Fuzzy control is easy to use because it usually does not require a mathematical model of the controlled system and operates very well in processes that are dynamic, poorly defined, non-linear. The benefit of fuzzy control is the use of human experience in the control process. Of course, one of the disadvantages of fuzzy controllers is that they are more complicated and complex to verify and prove their stability than traditional controllers, such as linear controllers or non-linear controllers.

Briefly in this paper, we aim to make a brief comparison between the QFT robust control method and the two control methods TFC and MFC, presented by prof. (Jing Lian & Fu Lin, 2005), in the Journal of Mechatronics.

## 2. Dynamic equations of the robotic manipulator

Fig.1 represents a two-degrees of freedom robot, where  $m_1, m_2$  are the masses of links 1, 2 and  $l_1, l_2$  are the lengths of each link respectively. The dynamic equation of the manipulator (Jing Lian & Fu Lin 2005) is presented as given below:

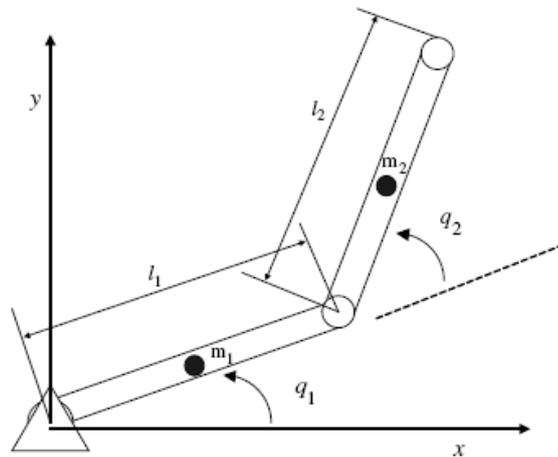


Figure 1. Two links robotic manipulator (Jing Lian & Fu Lin 2005)

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \quad (1)$$

with the mass matrix:

$$M(q) = \begin{bmatrix} \frac{1}{3}m_1 + m_2 l_1^2 + \frac{1}{3}m_2 l_2^2 + m_2 l_1 l_2 \cos q_2 & \frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 \cos q_2 \\ \frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 \cos q_2 & \frac{1}{3}m_2 l_2^2 \end{bmatrix}$$

$C(q, \dot{q})$  is a  $2 \times 2$  matrix of Coriolis and centrifugal forces, given as:

$$C(q, \dot{q}) = \begin{bmatrix} -\frac{1}{2}m_2l_1l_2(2\dot{q}_2) & -\frac{1}{2}m_2l_1l_2\dot{q}_2 \sin q_2 \\ \frac{1}{2}m_2l_1l_2\dot{q}_1 \sin q_2 & 0 \end{bmatrix}$$

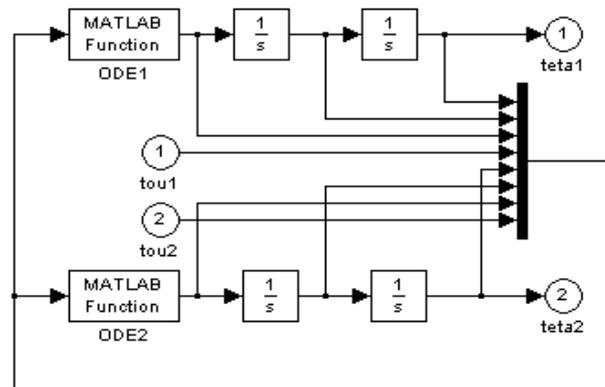
And  $G(q)$  is a  $2 \times 1$  gravity vector represented as:

$$G(q) = \begin{bmatrix} (\frac{1}{2}m_1 + m_2)gl_1 \cos q_1 + \frac{1}{2}m_2gl_2 \cos(q_1 + q_2) \\ \frac{1}{2}m_2gl_2 \cos(q_1 + q_2) \end{bmatrix}$$

where  $g$  depicts the gravity acceleration constant.

For an illustrative example the following numerical values are chosen for the robot manipulator ( $m_1=2\text{kg}$ ,  $m_2=3\text{kg}$ ,  $L_1=0.4\text{m}$ , and  $L_2=0.6\text{m}$ ) (Jing Lian & Fu Lin 2005)).

Block diagram representation of the above equations which simulate nonlinear multivariable dynamics of the robot in Matlab is shown in Figure 2.



**Figure 2.** Simulation of Robot Dynamic in Matlab

**2.1. Linearization**

In the process of linearization of the system dynamic equations, a part of the system specifications is ignored, and because most of these methods perform the linearization in the vicinity of the equilibrium point, there is always a question of how valid the deviation from the linearized point of work is. In practice, therefore, the compensator designed based on these methods is not responsive to the system due to lack of robustness, and most of these designs are theoretically carried out. The dynamics of the actual systems are generally unpredictable and unclear. The purpose of robust control is to control these systems. In the QFT method, which is one of the types of robust control methods, the nonlinear model of the system is converted to a certain number of linear transfer functions of the system with uncertainties.

As a result, the linearized transfer function for each link can be obtained as follows:

$$P_i = \frac{1}{s(J_{eff}s + C_{eff})} \quad : \quad i=1, 2 \tag{2}$$

For the first link:

$$J_{eff} = [5.3838 \ 7.3716] \text{ and } C_{eff} = [27.9061 \ 55.7397] \tag{3}$$

And for the second Link:

$$J_{eff} = [3.1258 \ 4.1913] \text{ and } C_{eff} = [-.3417 \ 17.4859] \tag{4}$$

Figure 3 depicts the structure of a two degrees of freedom system.

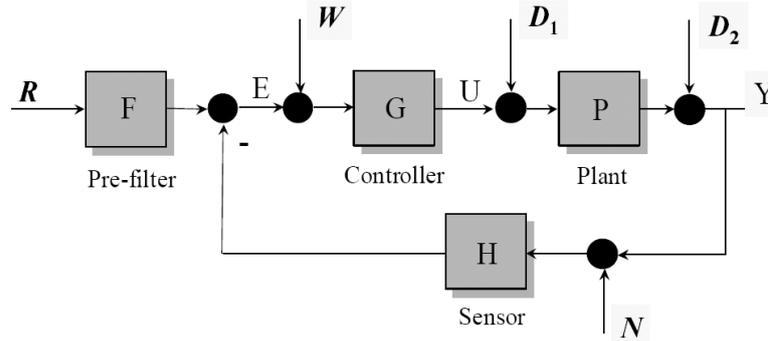


Figure 3. Structure of a Two Degrees of Freedom System

2.2. QFT Controller Design

The approach of the QFT system is very straightforward, enabling the designer to see the required trade-offs to meet the requirements of the closed-loop structure. After linearizing the system and obtaining the uncertainties of the system (the non-linear plant has to be transformed into a family of the linear and uncertain systems), template generation, as well as the nominal plant of the system, are selected. Then the tracking and stability bonds are determined. Finally, after obtaining the most critical performance specifications, the controller and pre-filter are designed. Finally, the system is modeled after the controller design, which indicates the approval of modeling and controller design (Nataraj, 2002), (Horowitz, 1992). To get the details of the controller design by robust control method, you can refer to the following references (Horowitz, 1991), (Horowitz, 1992), (Gharib & Moavenian, 2014), Jahanpour et al. (2015), (Gharib & Moavenian, 2016), (Gharib & Danshvar, 2019), Honari-Torshizi et al. (2020).

The appropriate QFT controller (G) and prefilter (F) (Fig.3) were then designed for the two links to conform with the closed-loop requirements ( $M_p=20\%$  and  $T_s=0.08$  s) where  $M_p$  and  $T_s$  are the overshoot and the settling time respectively)

Note: In order to save space, all steps of the controller design are shown only for the first link.

The template generation, the robust margin bonds for chosen trajectories, and the intersection of the boundaries of the first link based on the frequencies found in the generation of templates are shown in Figs.4, 5, and 6, respectively.

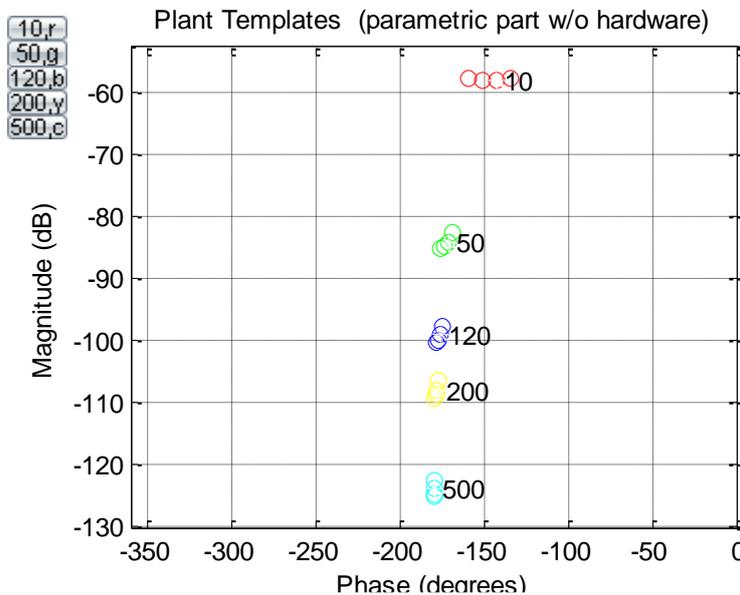


Figure 4. Template Generation of the first link

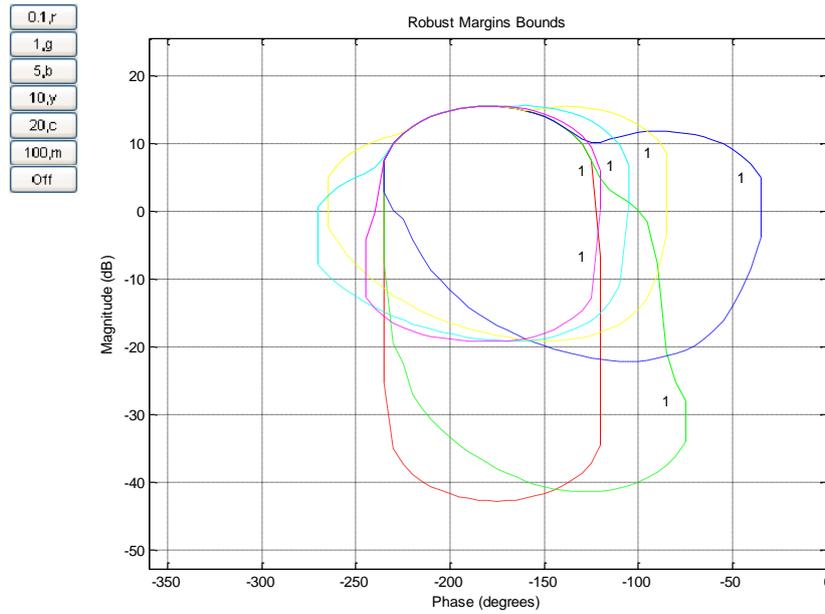


Figure 5. Robust Margin Bounds for the first link

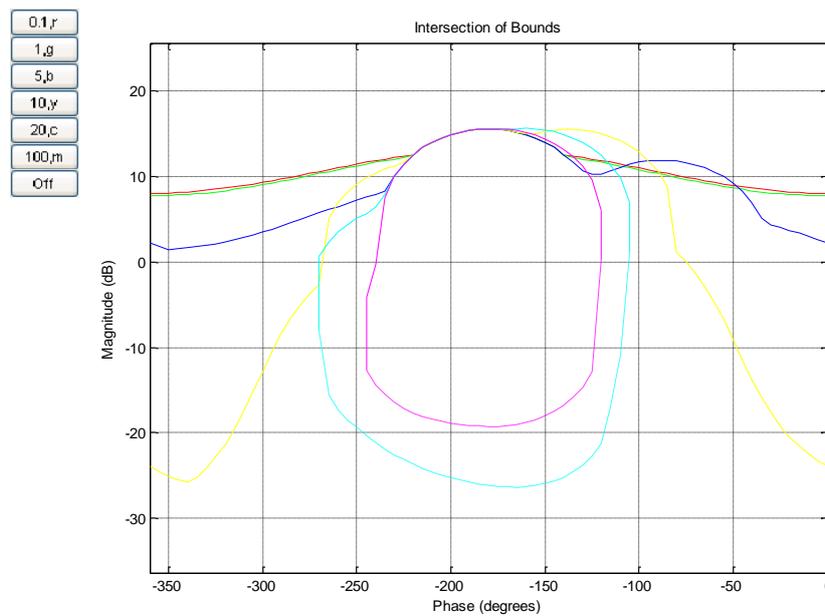


Figure 6. The intersection of the Bounds of the first link

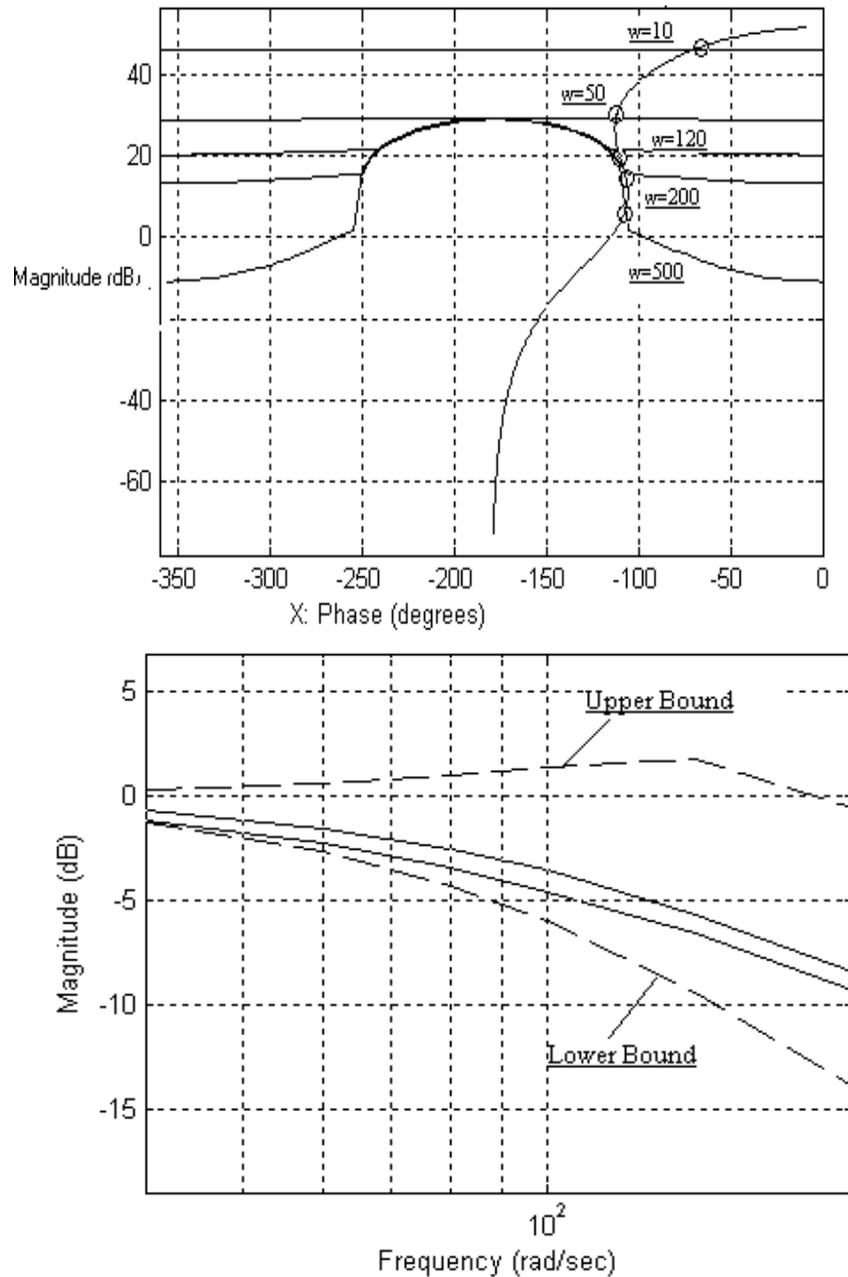
### 2.3. QFT Controller Design using GA

Evolutionary computing is the most effective computational intelligence method. This soft computing technique uses computational redundancy to form an efficient population of candidate solutions.

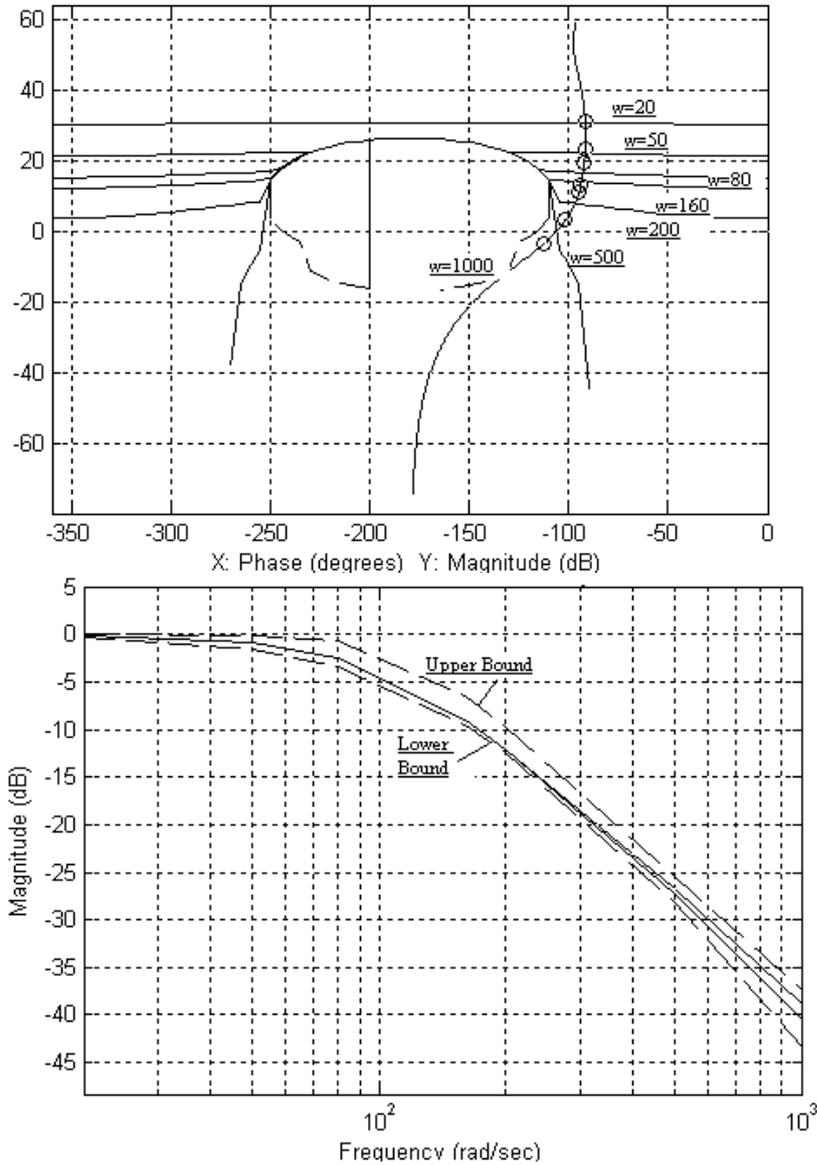
The Genetic Algorithm (GA) is the most representative evolutionary algorithm that can encode, and thereby maximize, the parameters and structures of the engineering solution. GA mimics human intelligence in learning and tuning based on trial-and-error. It implements the concept of self-'survival-of-the-fittest' selection and replication and does not require any teacher or gradient knowledge.

After replicating better-performing candidates, the GA then deviates from the search in an operation called 'crossover' by exchanging coordinates or parameters between surviving candidates. It also differs from the search by modifying certain parameter values in an operation called 'mutation'. 'In this way, a new 'generation' of candidate designs will be created and the emulated developmental loop will begin until no significant change in the design can be made.

In this study, an automated loop-shaping algorithm is used to merge the benefits of the conventional manual loop-shaping process with those of the GAs. The feature and/or benefit of the suggested system as follows is obtained from the manual loop-shaping method.



**Figure 7.** Loop-Shaping and Pre-Filter In Nichols Chart for the first link



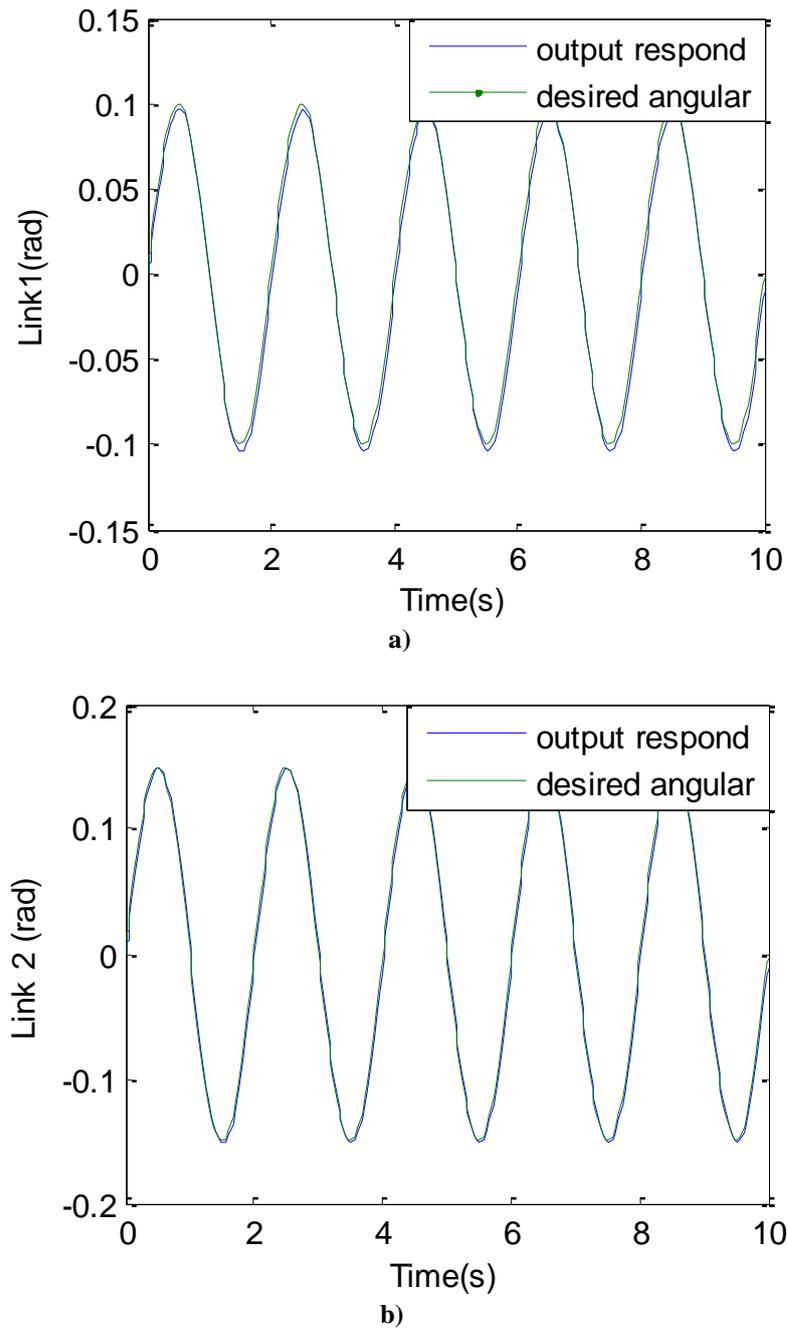
**Figure 8.** Loop-Shaping and Pre-Filter In Nichols Chart for the second link

The respected controller and prefilter for links (1) and (2) are found respectively as follow:

$$G(s) = 238.33 \frac{s(s+77.58)}{\left(\frac{s}{26.76} + 1\right)\left(\frac{s}{2213} + 1\right)} \quad F(s) = \frac{138.1^2 \left(\frac{s}{204.3} + 1\right)}{\left(\frac{s}{122.1} + 1\right)(s^2 + 207.15s + 138.1^2)} \quad (4)$$

$$G(s) = 3033.3 \frac{(s + 2.012)}{\left(\frac{s}{2444} + 1\right)} \quad F(s) = \frac{107.5^2 \left(\frac{s}{1542} + 1\right)}{(s^2 + 181.1s + 107.5^2)} \quad (5)$$

Angular tracking responses were used to evaluate the control performance of the robotic system. Fig. 9 plots the simulation angular tracking responses of this control system using the QFT controller.



**Figure 9.** Angular tracking responses using a QFT: (a) the first, and (b) the secondary link.

### 3. Fuzzy controller

#### 3.1. Introduction of fuzzy controller

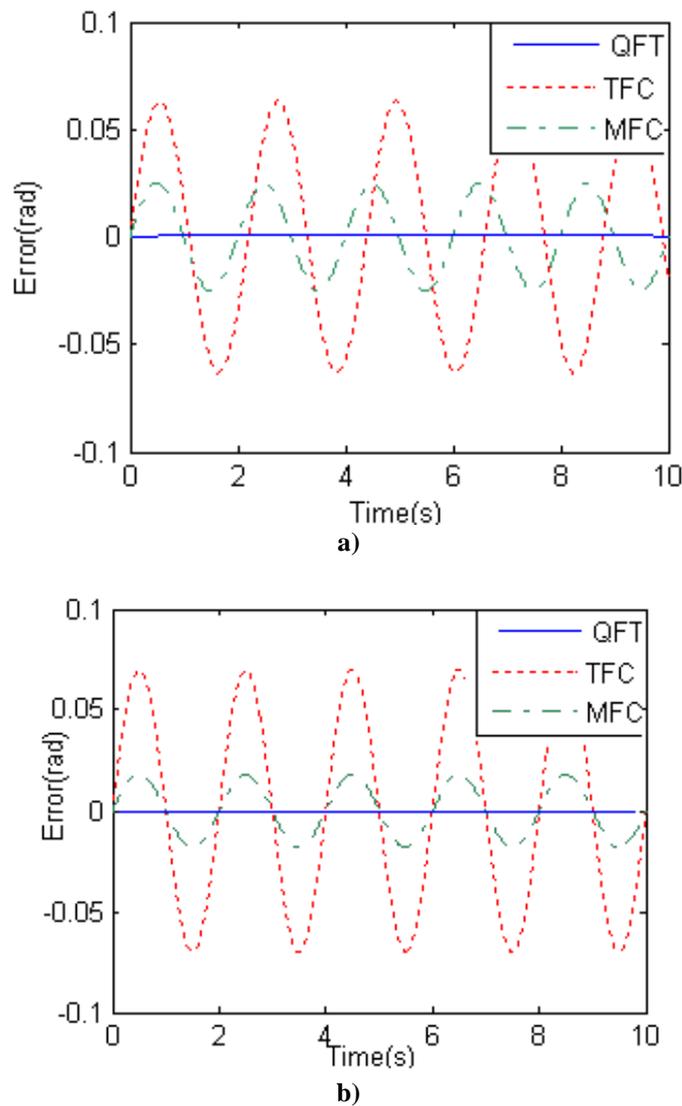
Many manufacturing processes managed by human operators cannot be programmed using traditional control methods because the operation of these controls is also not appropriate for operators. One explanation for this is that linear controllers, which are mostly used in traditional control systems, are not suitable for non-linear systems. Another reason is that person collects various knowledge and combines control methods that cannot be organized into a single analytical control rule. Because the fuzzy controller is an estimated rationale-based model lacking an analytical model for stability and robustness the industrial equipment implementation is hesitant. This issue can be resolved by adding a mixed fuzzy system (MFC) ((Jing Lian & Fu Lin 2005)), a sliding-mode control (Utkin VI. (1977)) and an adaptive fuzzy controller (Ching Chiou, K. (2005)).

**3.2. TFC and MFC controller**

To control a robot with a fuzzy controller in minimal error, designers require two fuzzy controllers. At first, for each link, only a TFC was programmed to regulate this MIMO robotic device. Second, a coupling fuzzy controller was added to the standard fuzzy control technique to enhance the control efficiency of this MIMO robotic system (Jing Lian & Fu Lin 2005).

**4. Results and Analysis**

In order to determine the appropriate control system, the GA-based QFT controller with a fuzzy methodology (Jing Lian, R. (2005)) is compared to the control robot with two links. Angular tracking responses have been used to analyze the control efficiency of the robotic system. Fig. 10 compares TFC, MFC, and QFT angular tracking errors.



**Figure 10.** Comparison of the TFC, the MFC, and the QFT methods for angular tracking errors (a) the first link and (b) the secondary link

Finally, Fig10 (a, b) compares the QFT method with the Fuzzy controller and shows that the QFT technique suggests a controller that has a better control performance (robustness, stability, tracking).

## 5. Conclusions

A practical method for developing a stable robot controller using quantitative feedback theory (QFT) is proposed in this study. The presence of uncertainty in the dynamics of robot arm manipulators ensures that the use of robust control methods to achieve high precision in tracking is necessary. QFT has been used to design a robust controller. Basic design phases can be summarized as the linearization of robot dynamics, the design of acceptable stable output limits by reducing the sensitivity function, linear simulation, and non-linear simulation.

A GA-based computer automatic modeling technique has been developed to overcome the QFT design problems encountered by a practical engineer. This can be used to easily provide the initial controller on which to base manual loop-shaping and filtering. This is especially beneficial in the case of unstable or non-minimum step plants or plants for which a stabilizing controller is difficult to find.

Also, the QFT approach is compared to the Fuzzy controller and it is shown that the QFT methodology indicates a controller that has increased control efficiency.

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## References

- Ching Chiou, K. & Shiu-Jer Huang, Sh. (2005). An adaptive fuzzy controller for robot manipulators, *Mechatronics* 15, 151–177.
- Gharib, M. R., & Daneshvar, A. (2019). Quantitative-fuzzy Controller Design for Multivariable Systems with Uncertainty. *International Journal of Control, Automation and Systems*, 17(6), 1515-1523.
- Gharib, M. R., & Moavenian, M. (2016). Full dynamics and control of a quadrotor using quantitative feedback theory. *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, 29(3), 501-519.
- Gharib, M. R., Dabzadeh, I., Mousavi, S. A. S., & Kamelian, S. (2010). Robust controller design with QFT and sliding mode for boiler pressure. In *Proceedings of the 2010 International Conference on Modelling, Identification and Control* (pp. 412-417). IEEE.
- Gharib, M. R., Kamelian, S., Seyyed Mousavi, S. A., & Dabzadeh, I. (2011). Modelling and multivariable robust controller for a power plant. *International Journal of Advanced Mechatronic Systems*, 3(2), 119-128.
- Gharib, M., & Moavenian, M. (2012). A new generalized controller for engine in idle speed condition. *Journal of Basic and Applied Scientific Research*, 2(7), 6596-6604.
- Gharib, M., & Moavenian, M. (2014). Synthesis of robust PID controller for controlling a single input single output system using quantitative feedback theory technique. *Scientia Iranica. Transaction B, Mechanical Engineering*, 21(6), 1861-1869.
- Honari-Torshizi, M., Rahmani, H., Moeinkhah, H., Gharib, M. R., & Jahanpour, J. (2020). A QFT robust controller as a remedy for TRMS. *Aviation*, 24(4), 137-148.
- Horowitz, I M. (1991). Survey of Quantitative Feedback Theory, *Int. J. Control Journal*, 33(2), 255-261.
- Horowitz, I. M. (1992). *Quantitative Feedback Design Theory (QFT)*. 1, QFT Publications, 4470 Grinnel Ave., Boulder, Colorado 80303, USA.
- Jahanpour, J., Honari-Torshizi, M., & Gharib, M. R. (2015). VCNC contour following tasks using robust QFT controller. *Iranian Journal of Science and Technology Transactions of Mechanical Engineering*, 39, 131-145.
- Jing Lian, R & Fu Lin, B. (2005). Design of a mixed fuzzy controller for multiple-input multiple-output systems, *Mechatronics*, 15, 1225–1252.
- Lewis, F.L. & Et, Al. (1999). *Robotics Mechanical Engineering Handbook*. Ed. Frank Kreith. Boca Raton: CRC Press LLC.

Luh, J. Y. S. (1983) An Anatomy of Industrial Robots and Their Controls, IEEE Trans. Automat. Contr., 28(2), 133-153.

Nataraj, P.S.V. (2002). Computation of QFT Bounds for Robust Tracking Specifications, Elsevier Science Ltd, Automatica, 38, 327-334.

Utkin, V.I. (1977). Variable structure systems with sliding mode: a survey. IEEE Trans Automat Contr; 22, 212-222.