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The Influence of Loading Position in A Priori High Stress Detection using Mode Superposition

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ABSTRACT

For the analysis of structural components, the finite element method (FEM) has become the most widely applied tool for numerical stress- and subsequent durability analyses. In industrial application advanced FE-models result in high numbers of degrees of freedom, making dynamic analyses time-consuming and expensive. As detailed finite element models are necessary for accurate stress results, the resulting data and connected numerical effort from dynamic stress analysis can be high. For the reduction of that effort, sophisticated methods have been developed to limit numerical calculations and processing of data to only small fractions of the global model. Therefore, detailed knowledge of the position of a component's highly stressed areas is of great advantage for any present or subsequent analysis steps. In this paper an efficient method for the a priori detection of highly stressed areas of force-excited components is presented, based on modal stress superposition. As the component's dynamic response and corresponding stress is always a function of its excitation, special attention is paid to the influence of the loading position. Based on the frequency domain solution of the modally decoupled equations of motion, a coefficient for a priori weighted superposition of modal von Mises stress fields is developed and validated on a simply supported cantilever beam structure with variable loading positions. The proposed approach is then applied to a simplified industrial model of a twist beam rear axle.

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1. Introduction

Just as the static strength, also the durability under dynamic loading is an essential property of structural components, as the unforeseen failure can cause catastrophic accidents or a malfunction (Schijve, 2001). Due to mass reduction and extensive exploitation of load bearing capacities, especially modern lightweight components from aerospace and automotive industry tend to be more sensitive to dynamic stress and failure (Rama et al., 2018) leading to increasing effort for numerical validation in the product development process. In addition to cost-intensive experimental investigations, in the last decades numerical methods, as the finite element method (FEM), have gained high importance in predicting stress fields of dynamically loaded structures. As detailed FE-models are necessary for sufficient stress accuracy, complexity of numerical models and numbers of degrees of freedom increase to a demanding task regarding disk space and CPU time. To overcome this, advanced methods are needed to meet the conflicting requirements of reducing computational cost and data as well as increasing stress accuracy. As direct methods can be very time consuming and costly, modal approximation methods are well-established in structural dynamics (Marinkovic & Zehn, 2018) in time domain as well as in frequency domain, which are based on modal decoupling of the system of equations of motion (Craig & Kurdila, 2006). With application of the so-called mode displacement method (MDM) computational cost can be reduced to only a fraction of the direct solution of the full system. More recently, modal stress methods have gained high importance, especially in durability and numerical fatigue analyses. From the modal displacement approach, Yam et al. (1996) derive the basic concept of strain modes and show, that just as the displacement response, also the strain response can be approximated by superposition of the contributions of the system's natural modes. Huang et al. (1998) apply modal transient methods for the durability analysis of a vehicle body structure, using modal stress superposition, as it is implemented in the FE-solver MSC.NASTRAN. Looking at the results of industrial stress analyses, one can see that typically high stress concentration is locally limited to critical regions of only a small fraction of the whole system. As indicated in the literature, the fraction of a model representing a fatigue concern is around 1% in automotive industry (Huang et al., 1997). For the identification of critical elements in (Huang et al., 1998), an element pre-scan is performed for elements exceeding a threshold value of von Mises stress in a short peak loading event. After this, a subsequent full stress history analysis and fatigue-life evaluation is limited to the top 100 elements. These investigations resulted in a patented method for identifying highly stressed elements (Huang & Agrawal, 2001). Gu et al. (Gu et al., 2012) investigate dynamic stress analysis of a mining dump truck from flexible multibody dynamics with modal representation of the structure, while stress recovery and durability analysis is performed, using stress mode superposition techniques. Lu et al. (2004) analyse the durability of a battery support system under base excitation using modal transient stress analysis and application of crack propagation calculations. Tran et al. (2013) present the analysis of crack propagation under dynamic loading using superposition of modal stress intensity factors as well as modal energy release rate with an extended FEM approach. For numerical fatigue analysis, Vellaichamy et al. (2000) modify the modal stress superposition, which is usually carried out as postprocess after modal transient analysis. First, displacement and stress modes are calculated in the FE-solver. Then, modal load is calculated with force vector and displacement modes, outside the FEA-environment. The transient results are then combined with the stress modes within the Fatigue solver. Mrnsik et al. (2018) describe a method for fatigue estimation of structures using a modal decomposition approach. The authors directly link the fatigue damage intensity with the dynamic properties of the system. The proposed method is based on modal decomposition of the system's dynamic stiffness matrix, giving insight to the modes' contribution to the system's total damage. Braccesi et al. (2016) evaluate the fatigue damage of structural components under random loading in the frequency domain using modal superposition. The method is based on the evaluation of the matrix of frequency response functions and stress mode superposition. For further reduction of computational effort, sophisticated methods have been developed to limit numerical calculations and processing of data to only critical regions of the global model. Albuquerque et al. (2015) propose a method based on the combination of finite element submodeling techniques and modal superposition using modal stress intensity factors. They demonstrate the applicability on a detailed FE-model of a critical detail of a railway bridge structure under transient traffic loading conditions. Lu et al. (2018) apply modal superposition techniques to the dynamic analysis of a high speed train bogie frame. Stress is calculated from a modal stress recovery method. With nominal stresses extrapolated from integration points, the contribution of each vibration mode to the fatigue damage at critical locations is investigated. Horas et al. (2016) describe a method for calculating a structure's fatigue life using superposition of modal stress intensity factors, derived from the system's eigenvectors and corresponding modal stress. The concept of modal superposition is extended to general fatigue damage quantities as stresses, strains or energetic parameters to assess local crack initiation, whereas detailed calculations are performed on local details of the global model.

The detailed knowledge of the position of a component's highly stressed areas is of great advantage for any present or subsequent analysis steps. Apart from established methods, which can generally be summarized as posteriori methods, as they root in the processing of existing stress data from pre-analysis steps, the system's information inherent in the eigenvectors and especially in the resulting stress mode shapes, can be applied for the development of powerful methods for a priori high stress detection. Despite the high potential benefit of the knowledge of highly stressed areas, only few research aims to these investigations. Veltri (2016) describes a method for the a priori prediction of stress concentration based on component-mode-synthesis (CMS) methods, namely the Craig-Bampton method, for the durability analysis of an all-terrain vehicle frame subjected to road-load time histories. Following the underlying theory of the Craig-Bampton reduction, information about highly stressed elements is gained from selected fixed interface normal modes. The influence of external loading is captured by a set of constrained modes. For numerical fatigue analysis of plate-type structures subjected to random base excitation, Zhou et al. (2016) propose a two-step procedure, where in a first step, highly stressed elements are located by means of modal stress analysis. In the second step, a fatigue analysis is carried out on these hot spots in the frequency domain. Later the contributions of dominant modes have been interpreted from mass modal participation factors of a notched elbow structure and a subsequent random vibration fatigue analysis has been performed on the identified hot spots (Zhou et al. 2017).

As the application of mass modal participation factors is theoretically limited to structures under base excitation, the procedure needs to be handled with care. In this paper a novel approach is developed, based on the a priori superposition of modal fields by means of appropriate approximation and prediction of maximum modal contributions for force excited systems. From the frequency domain solution of the modally decoupled equations of motion, a coefficient for a priori weighted superposition of modal von Mises stress fields is developed and validated on a simply supported cantilever beam structure with variable loading positions. The proposed approach is then applied to a simplified industrial model of a twist beam rear axle. The paper is structured as follows: In section 2, the basic equations of modal stress superposition are summarized and an approximation coefficient for a priori application is derived in section 3. For validation, investigations on a simple cantilever beam example are summarized, followed by application of the proposed approach to a simplified industrial example of a twist beam rear axle in section 4 and a summary and conclusion in section 5.

2. Modal Stress Approach

The finite element modeling of structural components for dynamic stress analysis results in a large number of second order differential equations (1). With the system's mass matrix **M**, damping matrix **C** and stiffness matrix **K**, the vectors of nodal accelerations $\mathbf{\ddot{u}}(\mathrm{t})$, nodal velocities $\mathbf{\dot{u}}(\mathrm{t})$ nodal displacements $\mathbf{u}(\mathrm{t})$ and the right-hand-side force vector **F**(t), the discretised system of equations is defined:

$$\mathbf{M\ddot{u}(t)} + \mathbf{C\dot{u}(t)} + \mathbf{Ku(t)} = \mathbf{F(t)} \tag{1}$$

From established approximation methods in structural dynamics it is well-known, that a component's dynamic displacement field can be approximated by linear superposition of the contributions of the system's eigenvectors $\boldsymbol{\varphi}_i$ and modal coordinates q_i (Craig & Kurdila, 2006). With the eigenvectors arranged as columns of the modal matrix **X**, the transformation of the system to modal coordinates is performed.

$$\mathbf{u}(\mathrm{t}) = \sum_{\mathbf{i=1}}^{\mathbf{r}} \boldsymbol{\varphi}_{\mathrm{i}} \mathrm{q}_{\mathrm{i}}(\mathrm{t}) = \mathbf{Xq}(\mathrm{t}) \tag{2}$$

Just as these displacement modes, also the so-called strain modes $\boldsymbol{\Psi}_{\mathrm{i}}$ are intrinsic dynamic characteristics of the vibrating structure. The strain modes are calculated as the derivative of the displacement modes. For the FE-discretised structure, this is achieved by a differential operator **D** applied to the displacement modes.

$$\boldsymbol{\Psi}_{\mathrm{i}} = \mathbf{D}\boldsymbol{\varphi}_{\mathrm{i}} \tag{3}$$

The structure's dynamic strain response $\boldsymbol{\varepsilon}$ can therefore be approximated by linear superposition of the contributions of modal strain fields (Yam et al., 1996).

$$\boldsymbol{\varepsilon} = \sum_{\mathrm{i=1}}^{\mathrm{r}} \mathrm{q}_{\mathrm{i}} \boldsymbol{\Psi}_{\mathrm{i}} \tag{4}$$

Assuming linear elastic and homogeneous material, the corresponding stress response $\boldsymbol{\sigma}$ is calculated using Hooke's matrix **H**.

$$\boldsymbol{\sigma} = \sum_{\mathrm{i=1}}^{\mathrm{r}} \mathrm{q}_{\mathrm{i}} \mathbf{H}\boldsymbol{\Psi}_{\mathrm{i}} \tag{5}$$

For consistent superposition and high stress localization, equivalent stress measures can be evaluated from the resulting modal stress fields on element level for each mode, e.g. the elemental von Mises stress:

$$\sigma_{\mathrm{eq}_{\mathrm{i,el}}} = \sqrt{\frac{1}{2}\left[\left(\sigma_{1,\mathrm{el}} - \sigma_{2,\mathrm{el}}\right)^2 + \left(\sigma_{2,\mathrm{el}} - \sigma_{3,\mathrm{el}}\right)^2 + \left(\sigma_{3,\mathrm{el}} - \sigma_{1,\mathrm{el}}\right)^2\right]} \tag{6}$$

Assuming modal damping, the equations of motion can be decoupled with the eigenvectors of the undamped system. Solving the system's eigenvalue problem, the i^{th} eigenvalue ω_i^2 and the corresponding eigenvector $\boldsymbol{\varphi}_i$ can be calculated.

$$(\mathbf{K} - \omega_{\mathrm{i}}^2 \mathbf{M})\boldsymbol{\varphi}_{\mathrm{i}} = \mathbf{0} \tag{7}$$

Modal mass m_i, modal damping c_i and modal stiffness k_i are then calculated from the system's mass, damping and stiffness matrix with corresponding eigenvectors:

$$\mathbf{X}^T\mathbf{MX} = \mathrm{diag}(\mathrm{m}_i) \quad ; \quad \mathbf{X^TCX} = \mathrm{diag}(c_i) \quad ; \quad \mathbf{X}^T\mathbf{KX} = \mathrm{diag}(\mathrm{k}_i) \tag{8}$$

By application of the modes' orthogonality properties, the modal coordinates q_i for the forced vibration problem can be evaluated from a set of decoupled second order differential equations, with the modal velocities $\dot{q}_i$ and modal accelerations $\ddot{q}_i$ (Craig & Kurdila, 2006).

$$\begin{bmatrix} \mathrm{m}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \mathrm{m}_\mathrm{r} \end{bmatrix} \begin{bmatrix} \ddot{\mathrm{q}}_1(\mathrm{t}) \\ \vdots \\ \ddot{\mathrm{q}}_\mathrm{r}(\mathrm{t}) \end{bmatrix} + \begin{bmatrix} \mathrm{c}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathrm{c}_\mathrm{r} \end{bmatrix} \begin{bmatrix} \dot{\mathrm{q}}_1(\mathrm{t}) \\ \vdots \\ \dot{\mathrm{q}}_\mathrm{r}(\mathrm{t}) \end{bmatrix} + \begin{bmatrix} \mathrm{k}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathrm{k}_\mathrm{r} \end{bmatrix} \begin{bmatrix} \mathrm{q}_1(\mathrm{t}) \\ \vdots \\ \mathrm{q}_\mathrm{r}(\mathrm{t}) \end{bmatrix} = \mathbf{X^TF}(\mathrm{t}) \tag{9}$$

The frequency domain solution for harmonic excitation $\mathbf{F}(t) = \hat{\mathbf{F}}e^{i\Omega t}$ can then be formulated for each row of equation (8).

$$\left(-\Omega^2 + 2\vartheta_i\omega_{0,i}i\Omega + \omega_{0,i}^2\right)\hat{q}_i e^{i\Omega t} = \frac{\hat{f}_i}{m_i}e^{i\Omega t} \tag{10}$$

With the modal load $\hat{f}_i$ for each mode,

$$\hat{f}_i = \boldsymbol{\varphi}_i^T\hat{\mathbf{F}} \tag{11}$$

the amplitude of the system's response in modal coordinates follows as

$$\hat{q}_i(\Omega) = \frac{1}{\left(-\Omega^2 + 2\vartheta_i\omega_{0,i}i\Omega + \omega_{0,i}^2\right)}\omega_{0,i}^2\frac{\hat{f}_i}{k_i}e^{i\Omega t} \tag{12}$$

By application of a modal stress approach and transforming back to physical coordinates, we find the complex stress transfer function $\mathbf{H}^\sigma(\eta)$ for the force-excited system with the frequency ratio $\eta_i = \Omega/\omega_{0,i}$

$$\mathbf{H}^{\sigma}(\eta) = \sum_{i=1}^{r}\mathbf{H}\boldsymbol{\Psi}_i\frac{\left((1-\eta_i^2)-i2\vartheta_i\eta_i\right)}{k_i\left(\left(1-\eta_i^2\right)^2+4\vartheta_i^2\eta_i^2\right)} \tag{13}$$

3. A Priori Mode Superposition

A mechanical component's dynamic response and corresponding stress is directly correlated to its excitation. The influence of loading position as well as loading direction therefore plays a crucial role in the development of an a priori mode superposition approach. From equation (12), we find the amplification function $V(\eta_i)$ for each mode:

$$V(\eta_i) = \frac{1}{k_i\sqrt{\left(1-\eta_i^2\right)^2+4\vartheta_i^2\eta_i^2}} \tag{14}$$

From the derivation of the amplification function of the underdamped system with $\vartheta \leq \frac{\sqrt{2}}{2}$, we find the maximum response at resonance:

$$\eta_{res} = \sqrt{1-\vartheta^2} \tag{15}$$

With the maxima of the amplification function from equation (14), the maximum modal response can be written as:

$$\hat{q}_i(t) = \frac{\boldsymbol{\varphi}_i^T\hat{\mathbf{F}}}{k_i 2\vartheta_i\sqrt{1-\vartheta^2}}e^{i\Omega t} \tag{16}$$

And, by application of equation (4), the maximum stress response can be approximated as

$$\boldsymbol{\sigma}_{max}(t) = \sum_{i=1}^{r}\mathbf{H}\boldsymbol{\Psi}_i\frac{\boldsymbol{\varphi}_i^T\hat{\mathbf{F}}}{k_i 2\vartheta_i\sqrt{1-\vartheta^2}}e^{i\Omega t} \tag{17}$$

Separation of the system's excitation to one spacial force-direction vector **f** and one time-dependent vector of loading functions **p**(t), then allows for a priori application of the modal stress superposition in non-harmonic loading. Loading position and direction in this case are captured by the dot-product of eigenvector and force-direction vector, independent from the excitation function.

$$\hat{f}_i = \boldsymbol{\varphi}_i^T\mathbf{f}\mathbf{p}(\mathbf{t}) \tag{18}$$

With the assumption that all modal maxima are in-phase, the solution can be seen as an upper bound for maximum dynamic stress and we find a weighting coefficient Γ_i for the a priori superposition of modal fields.

$$\Gamma_i = \frac{\boldsymbol{\varphi}_i^T\mathbf{f}}{k_i 2\vartheta_i\sqrt{1-\vartheta^2}} \tag{19}$$

As the maximum response is considered at resonance, frequency dependency is eliminated.

$$\boldsymbol{\sigma}_{max} = \sum_{i=1}^{r}\mathbf{H}\boldsymbol{\Psi}_i\Gamma_i \tag{20}$$

For validation of these assumptions, a simple example of a beam structure is investigated. The cantilever beam (see Figure 1) has a length of 1 m and a square cross-section of 10 x 10 mm². It is supported at its end and discretised with 100 linear beam elements. For investigation of the influence of the loading position on the modal contributions, the system's steady state response in a frequency range of 2 kHz is analyzed for 9 loading positions in y-direction.

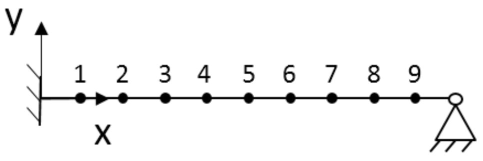

Figure 1. Cantilever beam example with 9 loading positions

The calculation is performed using modal decoupling with the system's first 10 eigenvectors and modal damping $\vartheta = 0.2$. The response of each modal coordinate for a unit load at position 1 is summarized in Figure 2.

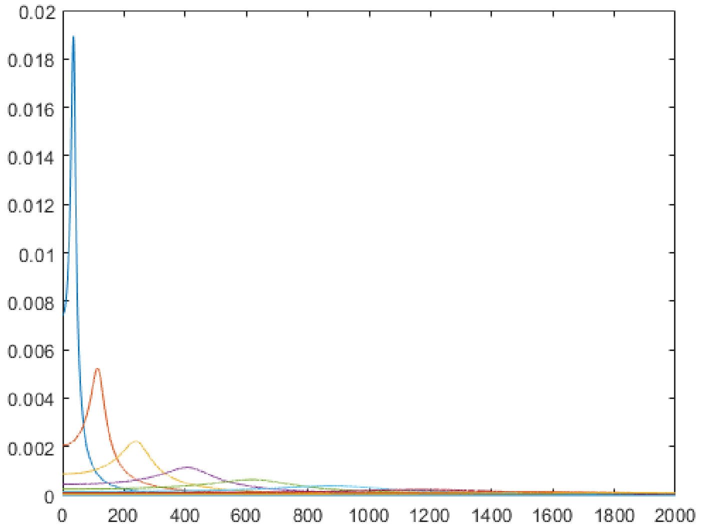

Figure 2. Response of the first 10 modal coordinates

For high stress detection, the maximum of each modal coordinate is taken for superposition of the corresponding modal fields as an upper bound reference solution, according to equation (19).The proposed weighting coefficient for a priori superposition in that case equals the maxima of the modal coordinates from the reference solution, as seen in Figure 3.

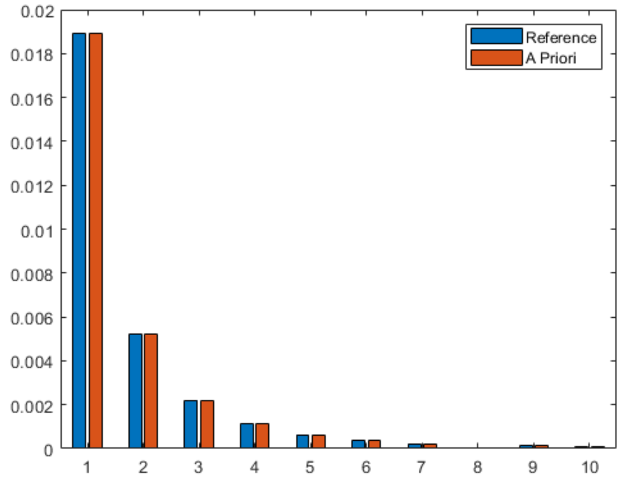

Figure 3. Comparison of modal contributions from reference and a priori approximation

As the coefficients of modal contributions are equal, both in reference solution and a priori approach, the resulting superimposed stress fields will consequently show no deviation, which will be shown in detail in the following application.

4. Industrial Application

To show applicability of the proposed approach to industrial models, a simplified beam model of a twist beam rear axle, seen in Figure 4, is investigated. For general investigations, excluding the effects of local details, the complex geometry of the real axle is defeatured and simplified to enable modeling with beam elements. The properties of the simplified model are adapted to meet the global dynamic properties of the real model in the lower eigenfrequencies with similar global mode shapes.

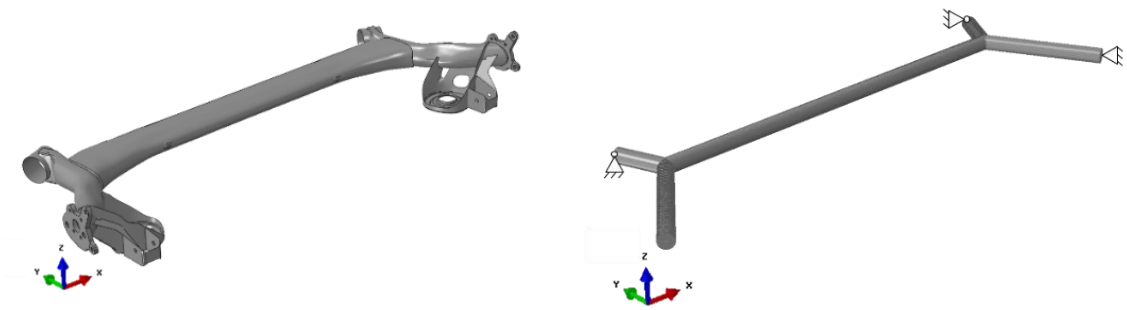

Figure 4. Industrial model and simplified beam model

The simplified model is discretised with 1876 linear beam elements of type B31, with an average element length of 1 mm and circular cross-section of 40 mm in diameter, material properties are depicted in Table 1. At the sleeve positions for axle bushings, boundary conditions are fixed, with free rotation around the local x-axis, for the areas of stub-axle assembly, one side is free and one side is fixed in both, displacement and rotation, as depicted in Figure 5.

Table 1. Material properties

Young's Modulus E [MPa]	Density ρ [t/mm³]	Poisson ratio ν [1]
220000	7.85	0.3

For investigation of the influence of the loading position on the resulting stress field, the system's steady state response for modal decoupling with the first 10 modes in a frequency range of 1000 Hz with a frequency step of $\Delta f = 1$ Hz and critical damping of $\vartheta = 0.2$ is analysed for 4 loading positions, as depicted in Figure 5.

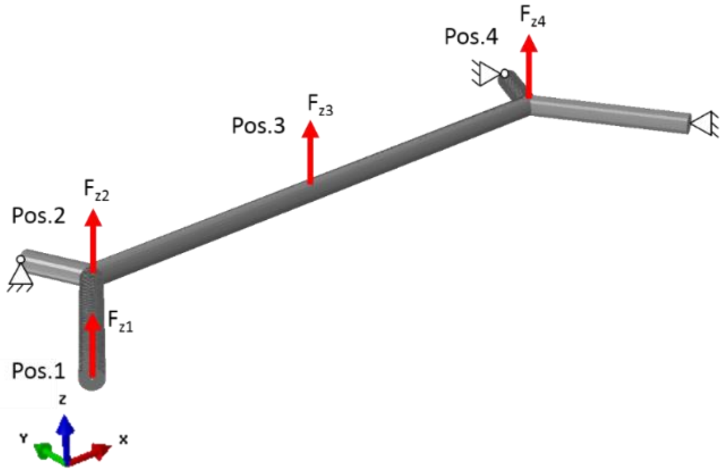

Figure 5. Boundary conditions and loading positions

As reference solution, the elemental von Mises stress is calculated from the superposition of the maxima of each modal coordinate with each corresponding stress mode, as derived in equation (16). The resulting stress over each element number is shown for all loading positions in Figure 6.

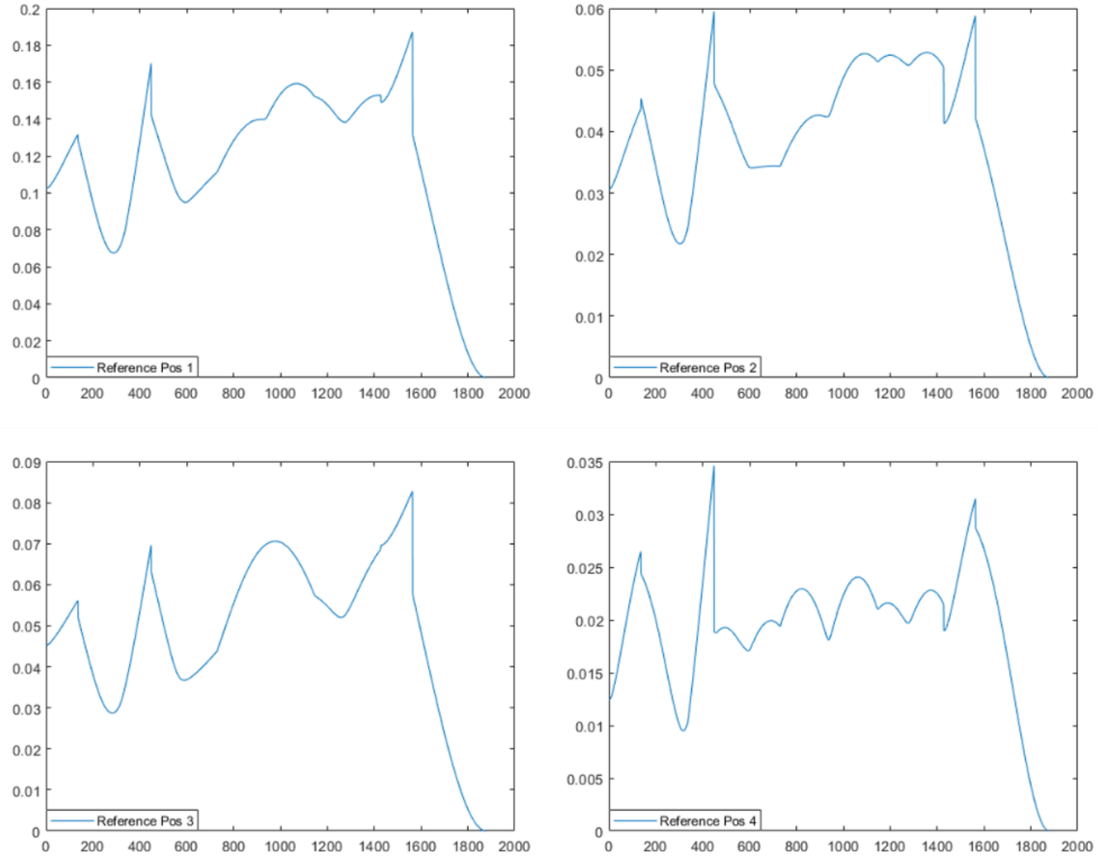

Figure 6. Von Mises Stress over element labels for reference solutions

The maximum modal contributions from the decoupled steady state analyses are summarized in Figure 7.

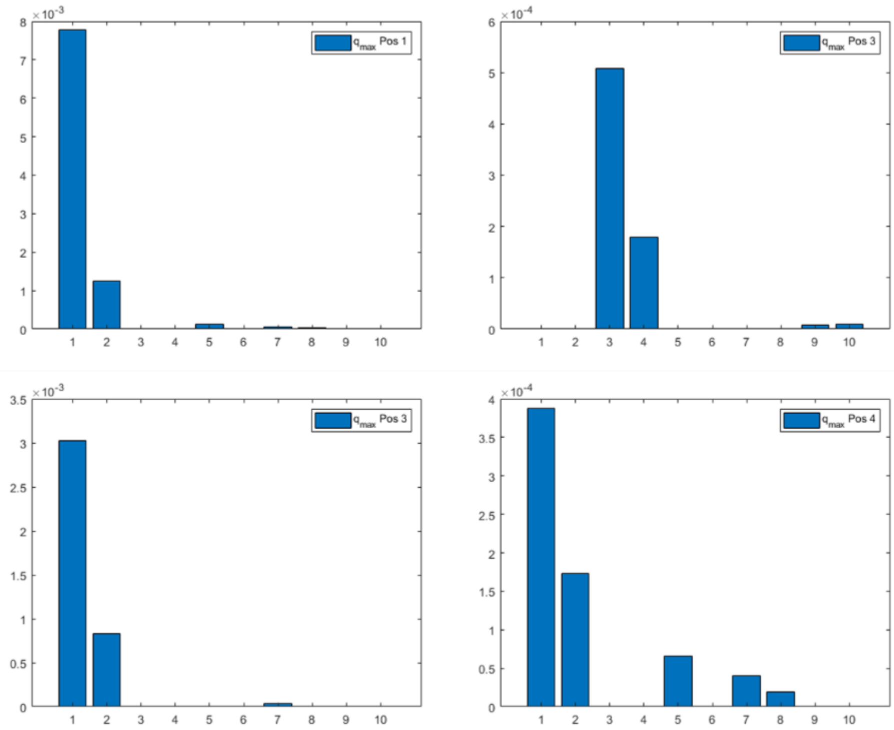

Figure 7. Maxima of modal coordinates for reference superposition

The results show dominant contributions of modes 1-4 for the four loading positions. For interpretation of the resulting superimposed stress fields, the von Mises stress of these dominant modes, resulting from mass-normalized eigenvectors, are depicted in Figure 8.

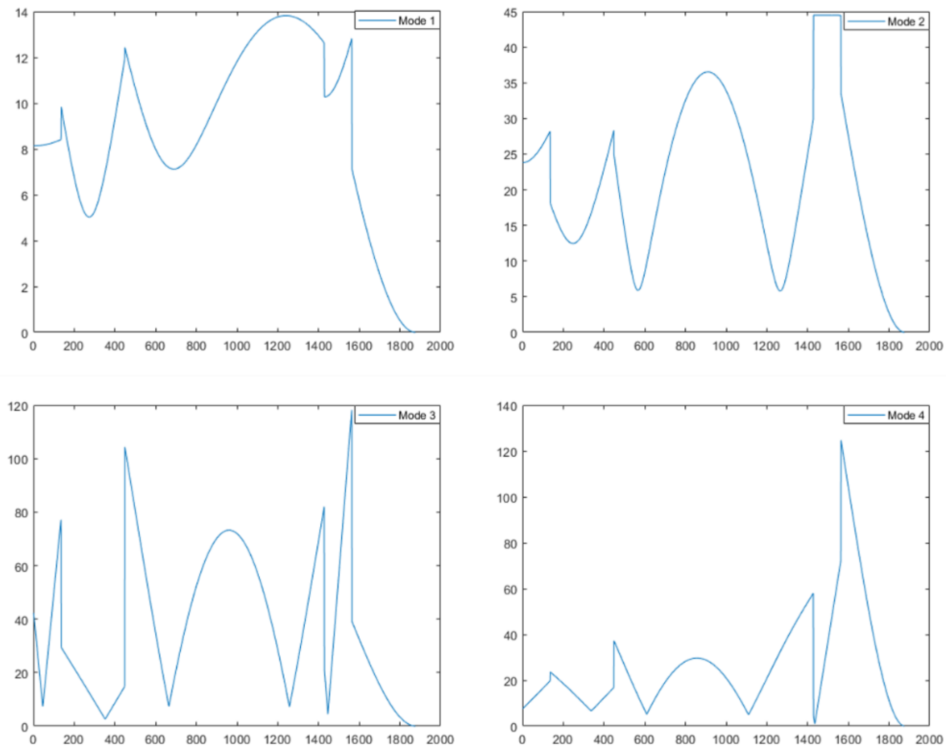

Figure 8. Von Mises stress modes for dominant modes 1-4

From Figures 7 and 8 we find plausible results for the overall superimposed stress field with maxima of modal coordinates and corresponding stress modes. Comparison of the reference solutions with the results the proposed a priori superposition method shows exact match of both solutions, as indicated in section 3.

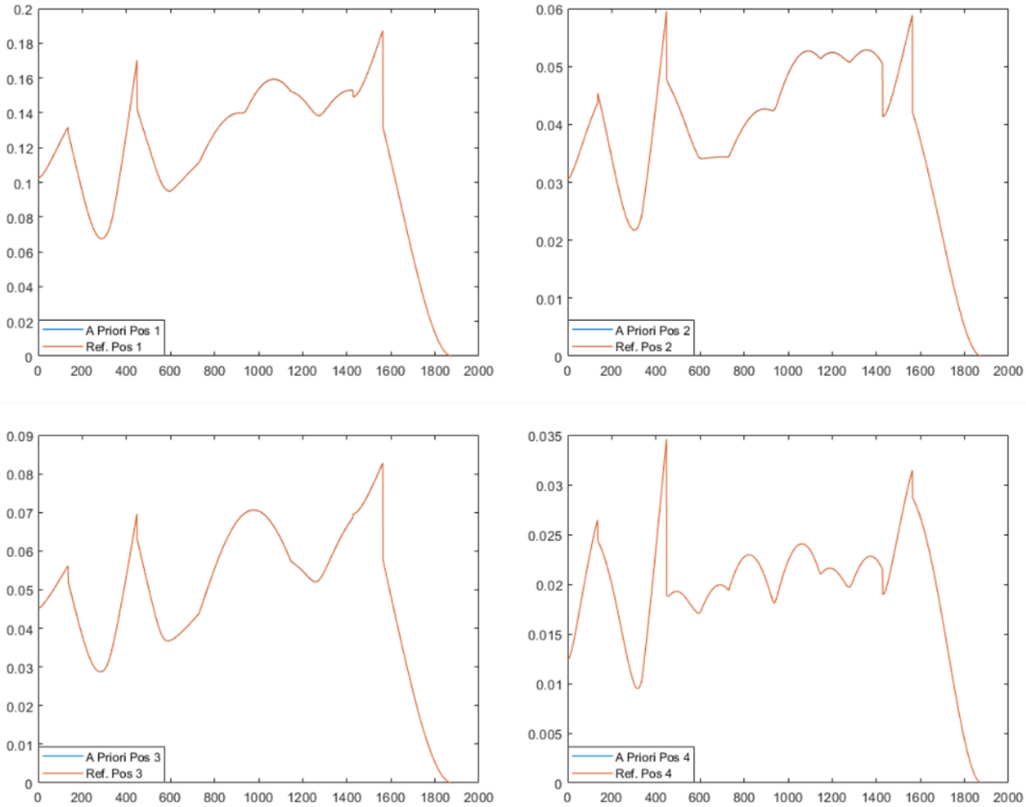

Figure 9. Comparison of reference solutions and a priori results (exact match)

5. Summary and Conclusion

In the presented paper, a method for a priori high stress detection has been proposed, based on modal stress superposition. From the frequency domain solution of the decoupled equations of motion, an analytically consistent weighting coefficient for a priori mode superposition has been developed. By separation of the loading function into one spacial term and one term accounting for time dependency, the influence of loading position and direction is captured prior to dynamic analysis. For validation, modally decoupled frequency domain analyses have been performed with variable loading positions. Application of the proposed approach to a complex model, adapted from automotive industry shows, that for harmonic excitation, the results exactly match the reference solution. From the superimposed stress field, valuable information about highly stressed areas, as well as uncritical elements can be gained. With that knowledge, sophisticated subsequent analysis steps can be limited to only fractions of the global model, e.g. submodeling, substructuring or crack propagation analyses, providing high potential for data reduction and increasing efficiency.

References

Albuquerque, C., Silva, A. L., de Jesus, A. M., & Calçada, R. (2015). An efficient methodology for fatigue damage assessment of bridge details using modal superposition of stress intensity factors. International Journal of Fatigue, 81, 61-77.

Braccesi, C., Cianetti, F., & Tomassini, L. (2016). An innovative modal approach for frequency domain stress recovery and fatigue damage evaluation. International Journal of Fatigue, 91, 382-396.

Craig, R. & Kurdila, A. J. (2006). Fundamentals of Structural Dynamics (2nd ed). Hoboken, New Jersey: John Wiley & Sons, Inc.

Gu, Z., Mi, C., Wang, Y., & Jiang, J. (2012). A-type frame fatigue life estimation of a mining dump truck based on modal stress recovery method. Engineering Failure Analysis, 26, 89-99.

Horas, C. S., Correia, J. A. F. O., De Jesus, A. M. P., Calçada, R., Aenlle, M. L., Kripakaran, P., & Fernandez-Canteli, A. (2016). Application of modal superposition technique in the fatigue analysis using local approaches. Procedia Engineering, 160, 45-5.

Huang, L., & Agrawal, H. (2001). U.S. Patent No. 6,212,486. Washington, DC: U.S. Patent and Trademark Office.

Huang, L., Agrawal, H., & Borowski, V. (1997). Durability Analysis of a Vehicle Body Structure Using Modal Transient Methods. In: Proc. SPIE Vol. 3089, Proceedings of the 15th International Modal Analysis Conference, 407–414.

Huang, L., Agrawal, H., & Kurudiyara, P. (1998). Dynamic Durability Analysis of Automotive Structures. SAE transactions, 474-480.

Lu, G. Y., Zheng, D., Venkatakrishnaiah, S., & Vest, T. (2004). A Dynamic Durability Analysis Method and Application to a Battery Support Subsystem (No. 2004-01-0874). SAE Technical Paper.

Lu, Y., Xiang, P., Dong, P., Zhang, X., & Zeng, J. (2018). Analysis of the effects of vibration modes on fatigue damage in high-speed train bogie frames. Engineering Failure Analysis, 89, 222-241.

Marinković, D., & Zehn, M. (2018). Consideration of stress stiffening and material reorientation in modal space based finite element solutions. Physical Mesomechanics, 21(4), 341-350.

Mršnik, M., Slavič, J., & Boltežar, M. (2018). Vibration fatigue using modal decomposition. Mechanical Systems and Signal Processing, 98, 548-556.

Rama, G., Marinkovic, D., & Zehn, M. (2018). High performance 3-node shell element for linear and geometrically nonlinear analysis of composite laminates. Composites Part B: Engineering, 151, 118-126.

Schijve, J. (2001). Fatigue of structures and materials. Dodrecht: Kluwer Academic Publishers.

Tran, V. X., Geniaut, S., Galenne, E., & Nistor, I. (2013). A modal analysis for computation of stress intensity factors under dynamic loading conditions at low frequency using extended finite element method. Engineering Fracture Mechanics, 98, 122-136.

Vellaichamy, S., & Keshtkar, H. (2000). New approaches to modal transient fatigue analysis. SAE transactions, 845-850.

Veltri, M. (2016). FEM Techniques for High Stress Detection in Accelerated Fatigue Simulation. In: Journal of Physics: Conference Series (Vol. 744, p. 012137).

Yam, L. H., Leung, T. P., Li, D. B., & Xue, K. Z. (1996). Theoretical and experimental study of modal strain analysis. Journal of Sound and Vibration, 191(2), 251–260.

Zhou, Y., Fei, Q., & Wu, S. (2017). Utilization of modal stress approach in random-vibration fatigue evaluation. In: Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, 231(14), 2603–2615.

Zhou, Y., Wu, S., Trisovic, N., Fei, Q., & Tan, Z. (2016). Modal Strain Based Method for Dynamic Design of Plate-Like Structures. Shock and Vibration.